

11. Verify Cayley-Hamilton theorem of the matrix A and hence find A<sup>-1</sup> where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

12. If  $\hat{a}$  and  $\hat{b}$  are Unit Vectors inclined at an angle 'A', then prove that

(i)  $\sin\left(\frac{A}{2}\right) = \frac{1}{2} |\hat{a} - \hat{b}|$

(ii)  $\cos\left(\frac{A}{2}\right) = \frac{1}{2} |\hat{a} + \hat{b}|$

(iii)  $\tan\left(\frac{A}{2}\right) = \frac{|\hat{a} - \hat{b}|}{|\hat{a} + \hat{b}|}$

13. Evaluate the following integrals :

(a)  $\int x^2 \sin x \, dx$

(b)  $\int x \sqrt{x+2} \, dx$

(c)  $\int \frac{x}{(x^2+2)(x^2+3)} \, dx$

A  
(21222)  
BCA-I Sem.

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Roll No. \_\_\_\_\_

**18005**

**B.C.A. Examination, Dec.-2022**

**MATHEMATICS - I**

**(BCA - 101)**

*Time : Three Hours ]*

*[Maximum Marks : 75*

**Note :** Attempt questions from **all** sections as per instructions.

**Section - A**

**(Very Short Answer Type Questions)**

**Note :** Attempt **all** questions of this section.

Each question carries 3 marks.

3×5=15

1. Give an example of matrices A, B such that AB=0 but A ≠ 0, B ≠ 0.

2. Verify Rolle's theorem for the function

$$f(x) = \sqrt{4-x^2} \text{ in the interval } [-2, 2].$$

3. Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2 \tan x}$ .

4. Write the relationship between Gamma and Beta function.

5. Find the characteristic roots of the matrix

$$A = \begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$$

### Section-B

#### (Short Answer Type Questions)

**Note :** Attempt any **two** questions out of the following three questions. Each question carries  $7\frac{1}{2}$  marks.

$$7\frac{1}{2} \times 2 = 15$$

6. Explain  $\log_e(1+x)$  in ascending powers of 'x'.

7. Solve by Cramer's Rule :

$$5x - 7y + z = 11$$

$$6x - 8y - z = 15$$

$$3x + 2y - 6z = 7$$

8. Prove that Conical text of given capacity will require the least amount of Canvas when the height is  $\sqrt{2}$  times the radius of the base.

### Section-C

#### (Long Answer Type Questions)

**Note :** Attempt any **three** questions out of the following five questions. Each question carries 15 marks.  $3 \times 15 = 45$

9. Trace the curve

$$4ay^2 = x(x-2a)^2$$

10. If  $y = (\sin^{-1} x)^2$  prove that

$$(1-x^2)y_2 - xy_1 - 2 = 0 \text{ also prove that}$$

$$(1-x^2)y_{n+2} - x(2n+1)y_{n+1} - n^2y_n = 0$$